nag_deviates_normal (g01fac)

1. Purpose

nag_deviates_normal (g01fac) returns the deviate associated with the given probability of the standard Normal distribution.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_deviates_normal(Nag_TailProbability tail, double p, NagError *fail)
```

3. Description

The deviate, x_p associated with the lower tail probability, p, for the standard Normal distribution is defined as the solution to:

$$P(X \leq x_p) = p = \int_{-\infty}^{x_p} Z(X)dX$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \quad -\infty < X < \infty.$$

The method used is an extension of that of Beasley and Springer (1977). p is first replaced by $q = p - 0.5$.

(a) if $|q| \leq 0.3$, x_p is computed by a rational Chebyshev approximation

$$x_p = s\frac{A(s^2)}{B(s^2)}$$

where $s = \sqrt{2\pi}.q$ and A, B are polynomials of degree 7.

(b) if $0.3 < |q| \leq 0.42$, x_p is computed by a rational Chebyshev approximation

$$x_p = \text{sign}\, q \left(\frac{C(t)}{D(t)}\right)$$

where $t = |q| - 0.3$ and C, D are polynomials of degree 5.

(c) if $|q| > 0.42$, x_p is computed as

$$x_p = \text{sign}\, q \left\{\left(\frac{E(u)}{F(u)}\right) + u\right\}$$

where $u = \sqrt{-2 \times \log(\min(p, 1-p))}$ and E, F are polynomials of degree 6.

For the upper tail probability $-x_p$ is returned while for the two tail probabilities the value x_{p^*} is returned where p^* is the required tail probability computed from the input value of p.

4. Parameters

tail

Input: indicates which tail the supplied probability represents.
If **tail** = **Nag_LowerTail**, the lower tail probability, i.e., $P(X \leq x_p)$.
If **tail** = **Nag_UpperTail**, the upper tail probability, i.e., $P(X \geq x_p)$.
If **tail** = **Nag_TwoTailSignif**, the two tail (significance level) probability, i.e., $P(X \geq |x_p|) + P(X \leq -|x_p|)$.
If **tail** = **Nag_TwoTailConfid**, the two tail (confidence interval) probability, i.e., $P(X \leq |x_p|) - P(X \leq -|x_p|)$.
Constraint: **tail** = **Nag_UpperTail**, **Nag_LowerTail**, **Nag_TwoTailSignif** or **Nag_TwoTailConfid**.

p

Input: the probability, p, from the standard Normal distribution as defined by **tail**.
Constraint: $0.0 < \mathbf{p} < 1.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

If **fail.code** $\neq$ **NE_NOERROR**, then nag_deviates_normal returns 0.0.

NE_BAD_PARAM
On entry, parameter **tail** had an illegal value.

NE_REAL_ARG_LE
On entry, **p** must not be less than or equal to 0.0: **p** = ⟨*value*⟩.

NE_REAL_ARG_GE
On entry, **p** must not be greater than or equal to 1.0: **p** = ⟨*value*⟩.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6. Further Comments

6.1. Accuracy

Accuracy is mainly limited by the ***machine precision***.

6.2. References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Ch. 7.1, p.297 and Ch. 26.2, p. 931 Dover Publications, New York.

Beasley J D and Springer S G (1977) Algorithm AS111. The Percentage Points of the Normal Distribution *Appl. Statist.* **26** 118–120.

Hastings N A J and Peacock J B (1977) *Statistical Distributions* Ch. 21, pp.96–101 Butterworth.

7. See Also

None

8. Example

Four values of **tail** and **x** are input and the probabilities calculated and printed.

8.1. Program Text

```
/* nag_deviates_normal(g01fac) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
```

```
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{

  double p;
  double dev;
  Integer i;
  char tail_char;
  Nag_TailProbability tail;

  Vprintf(" g01fac Example Program Results\n");
  /*     Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vprintf("\n  Tail    Probability     Deviate \n\n");
  for (i = 1; i <= 4; ++i)
    {
      Vscanf("%c %lf ", &tail_char, &p);
      switch (tail_char)
        {
        case 'L':
          tail=Nag_LowerTail;
          break;
        case 'U':
          tail=Nag_UpperTail;
          break;
        case 'C':
          tail=Nag_TwoTailConfid;
          break;
        case 'S':
          tail=Nag_TwoTailSignif;
        }

      dev = g01fac(tail, p, NAGERR_DEFAULT);
      Vprintf("   %c         %5.3f         %6.4f\n", tail_char, p, dev);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01fac Example Program Data
L 0.975
U 0.025
C 0.95
S 0.05
```

8.3. Program Results

```
 g01fac Example Program Results

  Tail    Probability     Deviate

   L         0.975         1.9600
   U         0.025         1.9600
   C         0.950         1.9600
   S         0.050         1.9600
```